List of Formulas

1. ***Trigonometry Identities***

$$\begin{aligned}\cos^2\theta + \sin^2\theta &= 1\\ 1 + \tan^2\theta &= \sec^2\theta\\ \cot^2\theta + 1 &= \csc^2\theta\end{aligned}$$

2. ***Double Angle Formula***

$$\begin{aligned}\sin(2\theta) &= 2\sin\theta\cos\theta\\ \cos(2\theta) &= \cos^2\theta - \sin^2\theta\\ \cos(2\theta) &= 2\cos^2\theta - 1 \quad \Longrightarrow \cos^2\theta = \frac{1+\cos(2\theta)}{2}\\ \cos(2\theta) &= 1 - 2\sin^2\theta \quad \Longrightarrow \sin^2\theta = \frac{1-\cos(2\theta)}{2}\end{aligned}$$

3. ***Some frequent derivative and Integral formulas***

$$(K)' = 0 \longleftrightarrow \int K\,dx = Kx + C \quad (K \text{ is a constant})$$

$$(e^x)' = e^x \longleftrightarrow \int e^x\,dx = e^x + C$$

$$(\ln x)' = \frac{1}{x} \longleftrightarrow \int \frac{1}{x}\,dx = \ln|x| + C$$

$$(x^n)' = nx^{n-1} \longleftrightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$(\tan\theta)' = \sec^2\theta \longleftrightarrow \int \sec^2\theta\,d\theta = \tan\theta + C$$

Product Rule: $(f\cdot g)' = f'\cdot g + f\cdot g'$

Chain Rule: $(f[g(x)])' = f'[g(x)]\cdot g'(x)$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$

L'Hospital Rule: $\text{“}\frac{0}{0},\ \frac{\infty}{\infty}\text{”} \Longrightarrow \lim\frac{f}{g} = \lim\frac{f'}{g'}$

Strategy for Integration

Firstly, try to simplify the integrand if possible. For example:

$$\int \sqrt{x}(1+\sqrt{x})\,dx = \int (\sqrt{x}+x)\,dx, \quad \int \frac{\tan\theta}{\sec^2\theta}\,d\theta = \int \frac{\sin\theta}{\cos\theta}\cos^2\theta\,d\theta, \cdots$$

Usually, the following **5** methods can cover all the integration problems.

1. ***Integration Formulas*:** Some elementary functions, for example:

$$\int x^n\,dx\,(n\neq -1), \int \frac{1}{x}\,dx, \quad \int \sin x\,dx, \int \sec x\,dx, \quad \int e^x\,dx, \quad \int \frac{1}{1+x^2}\,dx, \cdots$$

2. ***u*-*Substitution*:** Some function $\underline{u=g(x)}$ and $\underline{du=g'(x)dx}$ show up at the same time. For example:

$$\int \frac{x}{x^2-1}\,dx, \int \frac{x}{\sqrt{x^2-1}}\,dx, \quad \int \sin^m x\cos^n x\,dx, \quad \int xe^{x^2}\,dx, \quad \int \frac{\ln x}{x}\,dx, \cdots$$

3. ***Integration by parts*:** Usually $\underline{\text{two different types of functions}}$ show up at the same time. And one of $\underline{\text{them usually is the power of } x}$. e.g:

$$\int x\sin x\,dx, \int x\sin^m x\cos^n x\,dx, \quad \int x^2e^x\,dx, \quad \int x\ln x\,dx, \int \ln x\,dx, \cdots$$

4. ***Rational functions* $\frac{P(x)}{Q(x)}$:** The key method is $\underline{\text{partial fractions}}$. For this case, just be careful of the algebraic calculation.

5. ***Radicals*:** Usually there two types of questions in this case:

   (a) $\underline{\textbf{Trigonometric substitution:}}$ To deal with something like $\sqrt{\pm x^2 \pm a^2}$.

$$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx, \quad \int \frac{1}{\sqrt{x^2-a^2}}\,dx, \quad \int \frac{x^3}{\sqrt{x^2+a^2}}\,dx, \cdots$$

   (b) $\underline{\textbf{Rationalizing substitution:}}$ To deal with something like $\sqrt[n]{ax+b}$ or sometimes even for more general $\sqrt[n]{g(x)}$. For example,

$$\int \sqrt{\frac{1-x}{1+x}}\,dx, \quad \int x\sqrt[3]{4x+3}\,dx, \int x^2\sqrt{2+x}\,dx\cdots$$